

### Selected Financial Formulae

Purpose	Formula
<b>Basic Time Value Formulae</b>	
Future Value of a Single Sum	$FV = PV(1 + i)^N$
Present Value of a Single Sum	$PV = \frac{FV}{(1 + i)^N}$
Solve for N for a Single Sum	$N = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + i)}$
Solve for i for a Single Sum	$i = \sqrt[N]{\frac{FV}{PV}} - 1$
Present Value of an Ordinary Annuity	$PV_A = Pmt \left[ \frac{1 - 1/(1 + i)^N}{i} \right]$
Future Value of an Ordinary Annuity	$FV_A = Pmt \left[ \frac{(1 + i)^N - 1}{i} \right]$
Present Value of an Annuity Due	$PV_{Ad} = Pmt \left[ \frac{1 - 1/(1 + i)^{(N-1)}}{i} \right] + Pmt$
Future Value of an Annuity Due	$FV_{Ad} = Pmt \left[ \frac{(1 + i)^N - 1}{i} \right] (1 + i)$
Present Value of an Annuity Growing at a Constant Rate (g)	$PV_{GA} = \frac{Pmt_1}{i - g} \left( 1 - \left( \frac{1 + g}{1 + i} \right)^N \right)$
Future Value of an Annuity Growing at a Constant Rate (g)	$FV_{GA} = \left[ \frac{Pmt_1}{i - g} \left( 1 - \left( \frac{1 + g}{1 + i} \right)^N \right) \right] (1 + i)^N$
Holding Period Return (single period)	$HPR = \frac{P_1 + \sum \text{Cash Flows}}{P_0} - 1$

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Holding Period Return with Reinvestment (for multiple sub-period returns)	$HPR_{Reinvest} = \prod_{t=1}^N (1 + HPR_t) - 1$
<b>Basic Security Valuation Formulae</b>	
Dividend Discount Model (AKA, the Gordon Model)	$V_{CS} = \frac{D_0(1+g)}{k_{CS}-g} = \frac{D_1}{k_{CS}-g}$
Two-stage Dividend Discount Model Notes: This equation is too long for one line. $g_1$ = Growth rate during high growth phase. $g_2$ = Growth in constant growth phase after $n$ . $n$ = Length of high growth phase. Assume $g_1 > k_{CS}$ and $g_2 < k_{CS}$	$V_{CS} = \frac{D_0(1+g_1)}{k_{CS}-g_1} \left[ 1 - \left( \frac{1+g_1}{1+k_{CS}} \right)^n \right] + \frac{D_0(1+g_1)^n(1+g_2)}{k_{CS}-g_2} \frac{1}{(1+k_{CS})^n}$
Three-stage Dividend Discount Model Notes: $n_1$ = Length of high growth phase. $n_2$ = Periods until constant growth phase. $n_2 = n_1 + \text{length of transition phase.}$	$V_{CS} = \frac{D_0}{k_{CS}-g_2} \left[ (1+g_2) + \frac{n_1+n_2}{2}(g_1-g_2) \right]$
Earnings Model	$V_{CS} = \frac{EPS_1}{k_{CS}} + \frac{RE_1 \left( \frac{ROE}{k_{CS}} - 1 \right)}{k_{CS}-g}$
Constant Growth FCF Valuation Model $V_{Ops}$ = Value of Total Operations $V_{Debt}, V_{Pref}$ = Value of debt and preferred stock $V_{Non-Ops Assets}$ = Value of non-operating assets	$V_{Ops} = \frac{FCF_1}{k_{CS}-g}$ $V_{CS} = V_{Ops} - V_{Debt} - V_{Pref} + V_{Non-OpAssets}$
Sustainable growth rate Note: $b$ = retention ratio = $1 - \text{payout ratio}$ $r$ = return on equity	$g = br$
Value of a Share of Preferred Stock	$V_P = \frac{D}{k_P}$

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Value of a Bond on a Payment Date	$V_B = Pmt \left[ \frac{1 - 1/(1 + k_d)^N}{k_d} \right] + \frac{FV}{(1 + k_d)^N}$
Quoted Price of a Bond on a Non-Payment Date $V_{B,0}$ = Value of bond at last payment date $\alpha$ = The fraction of the current period that has elapsed	$V_{B,\alpha} = V_{B,0}(1 + k_d)^\alpha - \alpha(Pmt)$
Basic Statistical Formulae	
Arithmetic Mean (Average)	$\bar{X} = \frac{1}{N} \sum_{t=1}^N X_t$
Geometric Mean (used for averaging returns, growth rates, etc.)	$\bar{G} = \sqrt[N]{\prod_{t=1}^N (1 + R_t)} - 1$
Expected Value (Weighted Average)	$E(X) = \sum_{t=1}^N \rho_t X_t$
Variance	$\sigma_X^2 = \sum_{t=1}^N \rho_t (X_t - \bar{X})^2$
Standard Deviation	$\sigma_X = \sqrt{\sigma_X^2}$
Covariance	$\sigma_{X,Y} = \sum_{t=1}^N [\rho_t (X_t - \bar{X})(Y_t - \bar{Y})]$
Correlation Coefficient	$r_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$
Beta (Note: $M$ is the market portfolio, and $i$ is the security or portfolio)	$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2} = \frac{r_{i,M} \sigma_i \sigma_M}{\sigma_M^2}$

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<b>Portfolio Formulae</b>	
Expected Return of a Portfolio	$E(R_P) = \sum_{i=1}^N w_i R_i$
Variance of a 2-security Portfolio	Using the covariance: $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{1,2}$ or, using the correlation coefficient: $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 r_{1,2} \sigma_1 \sigma_2$
Variance of an N-security portfolio Using the Covariance	$\sigma_P^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{i,j}$
Standard Deviation of a Portfolio	$\sigma_P = \sqrt{\sigma_P^2}$
Portfolio Beta	$\beta_P = \sum_{i=1}^N w_i \beta_i$
95% Value at Risk (Variance/Covariance Model) Note: $V_p$ is portfolio value	$\text{VaR} = 1.645 \times V_p \times \sigma_p$
<b>Capital Market Theory Models</b>	
Capital Market Line (CML)	$E(R_P) = R_f + \sigma_P \frac{(E(R_M) - R_f)}{\sigma_M}$
Capital Asset Pricing Model (CAPM) Note: This is also the equation for the Security Market Line (SML)	$E(R_i) = R_f + \beta_i (E(R_M) - R_f)$
Treynor's Risk-adjusted Performance Measure	$T_i = \frac{R_i - R_f}{\beta_i}$
Sharpe's Risk-adjusted Performance Measure	$S_i = \frac{R_i - R_f}{\sigma_i}$

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Jensen's Alpha	$\alpha_i = \overline{(R_i - R_f)} - \beta_i \overline{(R_M - R_f)}$
The Information Ratio	$IR_P = \frac{R_P - R_B}{\sigma_{R_P - R_B}}$
M <sup>2</sup> (Modigliani & Modigliani) Performance Measure	$M^2 = \left(\frac{\sigma_m}{\sigma_i}\right)(R_i - R_f) + R_f$
Fama's Risk Decomposition Notes: R <sub>i</sub> = Portfolio Return R <sub>M</sub> = Market Return R <sub>f</sub> = Risk-free Rate β <sub>i</sub> = Portfolio Beta β <sub>T</sub> = Target Beta	<p>Risk Premium = R<sub>i</sub> - R<sub>f</sub></p> <p>Risk = β<sub>i</sub>(R<sub>M</sub> - R<sub>f</sub>)</p> <p>Selectivity = Risk Premium - Risk</p> <p>Managers Risk = (β<sub>i</sub> - β<sub>T</sub>)(R<sub>M</sub> - R<sub>f</sub>)</p> <p>Investors Risk = β<sub>T</sub>(R<sub>M</sub> - R<sub>f</sub>)</p> <p>Diversification = <math>\left(\frac{\sigma_i}{\sigma_M} - \beta_i\right)(R_M - R_f)</math></p> <p>Net Selectivity = Selectivity - Diversification</p>
Brinson, Hood, and Beebower Additive Attribution Model Notes: A <sub>t</sub> = Overall Allocation Effect S <sub>t</sub> = Overall Selection Effect I <sub>t</sub> = Overall Interaction Effect w <sub>i,t</sub> = Weight of Sector i in portfolio t bars over variables represent benchmark weights and returns.	$A_t = \sum_{i=1}^N (w_{i,t} - \bar{w}_{i,t})(\bar{R}_{i,t} - \bar{R}_t)$ $S_t = \sum_{i=1}^N \bar{w}_{i,t}(R_{i,t} - \bar{R}_{i,t})$ $I_t = \sum_{i=1}^N (w_{i,t} - \bar{w}_{i,t})(R_{i,t} - \bar{R}_{i,t})$
Options and Futures Valuation Models	
Black-Scholes European Call Option Valuation Model	$C = SN(d_1) - Xe^{-rt}N(d_2)$ <p>where:</p> $d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}$ $d_2 = d_1 - \sigma\sqrt{t}$

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Black-Scholes European Put Option Valuation Model (see above for $d_1$ and $d_2$ )	$P = Xe^{-rt}N(-d_2) - SN(-d_1)$
Put-Call Parity for European Options with No Cash Flows	$C = P + S - Xe^{-rt}$ or, $P = C + Xe^{-rt} - S$
Single-period Binomial Option Pricing Model for Call Options ( $r$ is the risk-free rate, $u$ is the up factor, and $d$ is the down factor)	$C = \frac{pC_u + (1-p)C_d}{(1+r)}$ where, $p = \frac{r-d}{u-d}$
Single-period Binomial Option Pricing Model for Put Options	$P = \frac{pP_u + (1-p)P_d}{(1+r)}$ where, $p = \frac{r-d}{u-d}$
Cost of Carry Model for Pricing Futures Contracts (CC is the carrying costs as a % of the spot price)	${}_T F_0 = S_0 e^{CC(t)}$
Bond Analysis Formulae	
Macaulay's Duration on a Payment Date (for immunization). Note: $C_t$ is the cash flow in period $t$ , $i$ is the yield to maturity	$D = \frac{\sum_{t=1}^N \frac{C_t(t)}{(1+i)^t}}{\text{Bond Price}}$
Modified Duration (for price volatility) on a Payment Date	$D_{Mod} = \frac{D}{(1+i)}$
Convexity on a Payment Date	$C = \frac{1}{(1+i)^2} \left[ \frac{\sum_{t=1}^N (t^2 + t) \frac{Cf_t}{(1+i)^t}}{\text{Bond Price}} \right]$
The $n$ -period forward rate given two spot rates (note that $i > j$ , and $n = i - j$ )	${}_{t+j}R_n = \sqrt[n]{\frac{(1+R_i)^i}{(1+R_j)^j}}$

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Bank Discount Yield for discount securities (FV = face value, PP = purchase price, m = periods per year)	$BDY = \frac{FV - PP}{FV} \times \frac{360}{m}$
Bond Equivalent Yield for discount securities (see definitions for BDY)	$BEY = \frac{FV - PP}{PP} \times \frac{365}{m} = BDY \times \frac{FV}{PP} \times \frac{365}{360}$
Capital Budgeting Decision Formulae	
Net Present Value ( <i>NPV</i> )	$NPV = \sum_{t=1}^N \frac{Cf_t}{(1+i)^t} - IO$
Profitability Index ( <i>PI</i> )	$PI = \frac{\sum_{t=1}^N \frac{Cf_t}{(1+i)^t}}{IO} = \frac{NPV + IO}{IO} = \frac{NPV}{IO} + 1$
Internal Rate of Return ( <i>IRR</i> ). Note: This is a trial and error procedure to find the <i>i</i> that makes the equality hold (i.e., what discount rate makes the <i>NPV</i> = 0).	$0 = \sum_{t=1}^N \frac{Cf_t}{(1+i)^t} - IO$
Modified Internal Rate of Return ( <i>MIRR</i> ).	$MIRR = \sqrt[N]{\frac{\sum_{t=1}^N Cf_t(1+i)^{(N-t)}}{IO}} - 1$
Stock Market Index Construction Formulae	
Price-weighted Average (e.g., DJIA) Note: The divisor (Div) at period 0 is equal to the number of stocks in the average. It will be adjusted for stock splits or any other corporate action that results in a non-economic change in the stock price.	$PWA_t = \frac{\sum_{j=1}^N P_j}{Div_t}$
Capitalization-weighted Index (e.g., S&P 500) Note: The divisor (Div) at period 0 is the divisor that makes the initial level of the index equal to the desired starting point. It will be adjusted for any corporate action that results in a change in market capitalization.	$CWI_t = \frac{\sum_{j=1}^N P_j Q_j}{Div_t}$

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Equally-weighted Arithmetic Index (e.g., VLA) Note: At period 0 the index is set to some starting value (e.g., 100). To calculate the index for any day, multiply the average % change by the previous index level.	$EWAI_t = EWAI_{t-1} \times \sum_{j=1}^N \left( \frac{P_{j,t}}{P_{j,t-1}} \right) / N$
Equally-weighted Geometric Index (e.g., VLG) Note: See note above	$EWGI_t = EWGI_{t-1} \times \sqrt[N]{\prod_{j=1}^N \frac{P_{j,t}}{P_{j,t-1}}}$
Corporate Financial Formulae	
Net Operating Profit After Taxes (NOPAT)	$\text{NOPAT} = \text{EBIT}(1 - t)$
Net Operating Working Capital (NOWC)	$\text{NOWC} = \text{Op. C.A.} - \text{Op. C.L.}$
Operating Capital (Op. Cap.)	$\text{Op. Cap.} = \text{NOWC} + \text{NFA}$
Free Cash Flow (FCF)	$\text{FCF} = \text{NOPAT} - \text{Net Investment in Op. Cap.}$
Economic Value Added (EVA)	$\text{EVA} = \text{NOPAT} - (\text{Op. Cap.} \times \text{Cost of Cap.})$
Beta of a Leveraged Firm	$\beta_L = \beta_U [1 + (1 - t)(D/S)]$
MM Value of Firm, No Corporate Taxes	$V_L = V_U = S_L + D$
MM Value of Firm With Corporate Taxes	$V_L = V_U + tD$
Merton Value of Firm with Personal Taxes	$V_L = V_U + \left[ 1 - \frac{(1 - t_C)(1 - t_S)}{(1 - t_D)} \right] D$
Miscellaneous Formulae	
Margin Call Trigger Price Note: IM% is the initial margin supplied, MM% is the maintenance margin requirement, P <sub>0</sub> is the initial value of the portfolio	$P_M = \frac{\text{IM}\% - 1}{\text{MM}\% - 1} \times P_0$
Percentage gain to recover (% GTR) from a loss (%L)	$\% \text{GTR} = \frac{1}{1 - \%L} - 1$