Teacher Education Does Matter: A Situative View of Learning to Teach Secondary Mathematics

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Abstract

The visions of mathematics classrooms called for by current educational reform efforts pose great challenges for K-12 schools and teacher education programs. While a number of colleges and universities throughout the country are making changes in their teacher education programs to reflect these reform recommendations, we have little systematic information on the nature of these programs or their impact on prospective teachers. These issues are of central concern in the study—Learning to Teach Secondary Mathematics in Two Reform-Based Teacher Education Programs (LTSM)—that we draw upon in this article. The article focuses on one pre-service teacher's (Ms. Savant) knowledge, beliefs, and practices related to proof, tasks, and discourse. A situative perspective on cognition and components of teachers' professional knowledge frame our research. We examined data on Ms. Savant's experiences in her teacher education program to understand of the influences of teacher education on her development as a mathematics teacher. This research indicates that Ms. Savant's teacher education experiences did make a difference in her development as a teacher. Her mathematics methods course provided a large collection of *tasks*, engaged her and her pre-service colleagues in *discourse,* and provided her with both formal and informal experiences with *proof*—all of these experiences reflecting reform-based visions of mathematics classrooms. The situative perspective on cognition directed our attention to issues of compatibility of goals and visions across the various university and K-12 classroom settings, and it helped us to understand why some aspects of reform-based pedagogy are more easily learned than others—why some ideas and practices learned as a student in the university setting are more easily transported to the novice teacher's K-12 field setting. We conclude that compatibility of these settings on several key dimensions is essential for the settings to reinforce each other's messages, and thus work in conjunction, rather than in opposition, to prepare reform-minded teachers.

Introduction

The visions of classrooms called for by current educational reform efforts pose great challenges for mathematics teachers and the schools in which they work. They represent a substantial departure from the K-12 classrooms in which most of today's teachers were students, and they are based on fundamentally different assumptions about mathematics, about teaching and learning, and about students and schools. To move successfully toward these visions requires major changes in many teachers' professional knowledge and beliefs, as well as their pedagogical practices.

The challenges to programs that prepare teachers are also great. Within the mathematics education community, for example, pre-service teacher education programs are being called upon to provide models of good mathematics teaching, help teachers develop their knowledge of mathematics and mathematics-specific pedagogy, provide multiple perspectives on students as learners of mathematics, and provide opportunities for teachers to develop their own identities as teachers of mathematics (NCTM, 1991). While a number of colleges and universities throughout the country are making changes in their teacher education programs to take reform-based visions of classrooms into account, we have little systematic information on the nature of these programs or their impact on prospective teachers. Further, despite the fairly extensive literature on teacher education and learning to teach, there continues to be much disagreement about the nature and extent of influence that teacher education and its various components have on teacher learning (Glickman & Bey, 1990; Grossman et al., 1999).

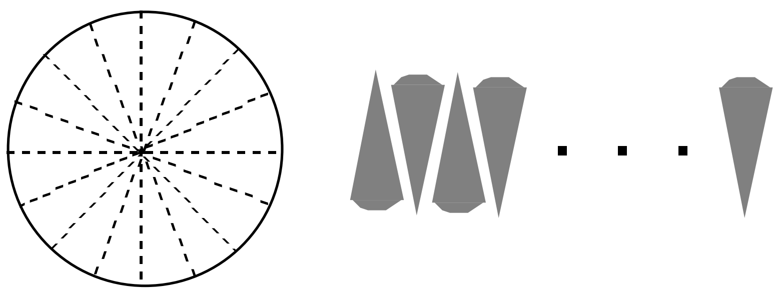
These issues are of central concern in the study—Learning to Teach Secondary Mathematics in Two Reform-Based Teacher Education Programs (LTSM) [1]—that we draw upon in this article. The article focuses on one pre-service teacher's knowledge, beliefs, and practices related to three key aspects of mathematics and mathematics-specific pedagogy. We begin by recounting a teaching episode in which Audrey Savant (all names and locations used in this article are pseudonyms), a student teacher, guided students to derive the formula for the area of a circle. We then outline the situative perspective on cognition

[1] Learning to Teach Secondary Mathematics in Two Reform-Based Teacher Education Programs (LTSM) is a multi-year research project, funded in part by the National Science Foundation (NSF Grant #REC9605030), in which we are

and the components of teachers' professional knowledge that frame our research. Next, we trace back through data on Ms. Savant's experiences in her teacher education program, in order to understand the influences of teacher education on that episode and, more generally, on her development as a mathematics teacher (See Appendix A for a list of data sources). We conclude by discussing the impact of teacher education on learning to teach.

Pizza π: Proving The Area Of A Circle

"Once you have your circle cut out, fold it so you get 16 pizza slices." Ms. Savant, the student teacher in charge of today's Algebra I lesson, calls out instructions as she demonstrates the first step of the task. Students, seated in groups of 3 or 4 at tables around the room, follow along. "As you get your slices cut out ... you're going to set them next to each other so that one faces down and one faces up, the next faces down, etc." After a few minutes, students have their wedges spread out on their desks, and Ms. Savant has a drawing on the white board at the front of the room:



This is the second day of a 4-day "mini-unit" on circles, within a larger text chapter on direct variation. Ms. Savant planned for students to do this activity to "see if they can prove [the formula for the area of a circle] using those wedges."

For the next few minutes, Ms. Savant questions the class about the dimensions of the figure created by this array of pizza slices. Pointing to the left-hand edge of her figure she asks, "What does this length represent on the circle?" One student, who comes to the board at Ms. Savant's prompting, explains by pointing to the edge of a pizza slice on the circle that its length is the radius of the circle.

Ms. Savant steps to the board, draws a bracket along the top of her figure, and asks "So, along here...what's this length, with all these curves?" She steps away from the board, waiting for a response.

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following prospective teachers from two reform-based secondary mathematics teacher education programs through their

Several students call out suggestions, including “circumference” and “half the circumference.” Ms. Savant asks the class, “OK, I’ve got half of the circumference, and I’ve got all of the circumference. What do you guys think?” A student goes to the board, traces over the arcs that form the top edge of the figure, and points to the corresponding arcs on the circle, explaining, “You have 8 of them (arcs) on the top, so it’s half.”

Ms. Savant prompts students to recall the formula for circumference of a circle, $C = 2\pi r$, a focus of the previous day’s class. Then, using a set of more and more directed questions, she guides them through algebraic manipulations to see that they can substitute πr for (1/2)C. They label the top edge of the pizza slices figure πr.

Now, the punch-line of this activity. Ms. Savant asks the class, “What kind of figure does this look like, when you lay it all out?” Several students respond, “Like a rectangle.” Ms. Savant repeats, “Like a rectangle,” and asks, “How would I find the area of the rectangle, Jordan?” He replies, “Length times width.” “OK,” she continues, “what’s the length of this rectangle (pointing to the figure)?” She waits several seconds before a student replies, “πr.” “OK, and what’s the width?” Someone calls out “r.”

Ms. Savant pauses a moment and asks, “So, to estimate this area (referring to the figure again), could I multiply these two together?” A student replies, “Yeah.” Ms. Savant concludes, “You guys have seen this formula before (writing the formula on the board as she talks), $A= \pi r^2$. So, what you have just done is taken a formula that you already knew and proved that it works.... You have just proved that the area of a circle is πr^2. You cut up the area, you rearranged it, and you proved that the area is πr^2. That’s great!”

This episode, particularly her use of the word “proof,” raised many questions for our research team. What was Ms. Savant’s understanding of proof? What did she believe about the role of proof in the secondary mathematics curriculum? Where did she get the idea for this activity, and why did she select it? How, if at all, did her experiences in teacher education affect her choice of this activity and her way of using it in class? Questions such as these are the focus of the analyses featured in this article. Before presenting these analyses, we describe the two components of the conceptual framework that ground the LTSM project and guide our efforts to understand teacher learning: a situative pespective on cognition and domains of teacher knowledge.

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last two years of teacher education and into their first two years of teaching.

A Situative Perspective On Teacher Cognition

For Ms. Savant, like other teachers, learning occurs in many situations of practice. Taking into consideration both teacher education and the induction years of teaching, these contexts include (but are not limited to) university mathematics and teacher-preparation courses, pre-service field experiences, and schools of employment. With its emphasis on the relationship between knowledge and the situations in which it is acquired and used, a situative perspective offers a compelling framework for the study of teacher learning. This perspective, drawn from recent work in cognitive science (Cobb & Bowers, 1999; Greeno, Collins & Resnick, 1996; Greeno & MMAP, 1998; Putnam & Borko, 1997, 2000) argues that one must study cognition and learning within the contexts in which they occur, taking into account both the individual learner and the physical and social systems in which that learner participates.

Traditional cognitive perspectives such as information processing theories (which we will refer to simply as cognitive perspectives), typically treat knowing as the manipulation of symbols inside the mind of the individual. Similarly, cognitive perspectives typically describe learning as an individual's acquisition of knowledge, change in knowledge structures, or growth in conceptual understanding. Context is not considered to be irrelevant to learning. Cognitive theorists argue, however, that while some learning takes place in a social context (e.g., on-the-job training), what is learned can also be independent of the context in which it is learned (Anderson et al., 1997).

Situative perspectives, in contrast, assume that knowledge is inseparable from the contexts and activities in which it develops. Challenging the view that there is an internal cognitive core independent of context and intention, situative perspectives posit that the physical and social context in which an activity takes place is an integral part of the activity, and that the activity is an integral part of the learning that takes place within it. Thus, how a person learns a particular set of knowledge and skills, and the situation in which a person learns, become a fundamental part of what is learned (Putnam & Borko, 1997, Resnick, 1991).

Some scholars contend that a situative perspective implies that the social collective or activity system is the principal unit of analysis (Greeno & MMAP, 1998; Lave , 1988). Indeed, it has been argued that individual versus social unit of analysis is a key distinction between cognitive and situative perspectives (Anderson et al., 1997, Lave, 1988). Cobb and colleagues disagree with this characterization, arguing that “the situated perspective admits a range of units of analysis, the choice in any particular case being a pragmatic one that depends on the purposes at hand” (Cobb & Bowers, 1999, p. 6). In a teaching experiment designed to facilitate and investigate students’ mathematical development within the social context of a third grade classroom, these researchers documented both the development of individual students’ place value conceptions and the evolution of the communal mathematical practices in which they participated. Further, they demonstrated that the relationship between classroom practices and individual students’ reasoning is reflexive. Students contribute to the development of practices within the classroom community; these practices, in turn, constitute the immediate context for their learning. Both social and individual units of analysis are important to understanding the learning that occurs in classroom communities: “an analysis of classroom mathematical practices documents the evolving social context of the students’ mathematical development, and an accompanying psychological analysis of the students’ activities as individuals documents the diverse ways in which they participated in those practices” (Bowers, Cobb & McClain, 1999, pp. 28-29). This research demonstrates the theoretical and practical contributions that a situative perspective—one that avoids the false dichotomy of individual cognition versus participation in social context—offers to the study of classroom practice. It’s theoretical contribution—a process model of the classroom community’s mathematical learning—led to practical contributions such as revisions to instructional materials.

We could find no comparable situative analyses of classrooms with teachers and teaching as their central focus. Perhaps the most impressive consideration of teaching from a cognitive perspective is the work of Schoenfeld and his Teacher Model Group to develop a theory of teaching-in-context (Schoenfeld, 1998). Schoenfeld and colleagues attempt, through the use of cognitive modeling strategies, to explain teachers’ decisions at each point of instruction by identifying the goals, beliefs, knowledge, and action plans on which those decisions are based. In an invited critique of this work, Greeno (1998) acknowledged that Schoenfeld and colleagues “provide a significant extension of cognitive theory and an important advance in the scientific understanding of teachers’ activity” (p. 111). He suggested, however, that a situative perspective would add significantly to this analysis by examining features of classroom social practices such as: patterns of discourse, the kinds of participation that are afforded to teacher and students by the classroom practices that are in place, and the personal identities developed by teacher and students through participation in these practices.

Similarly, a situative perspective provides a compelling framework for the study of teacher learning. The novice teacher's learning-to-teach experiences can be conceptualized as a single learning trajectory through the multiple contexts of teacher education. A situative perspective offers a way of disentangling—without isolating—the complex contributions of these various contexts to novice teachers' development. Before using this perspective to examine Ms. Savant's teaching in the Pizza Slices activity, we describe the components of professional knowledge that are the focus of our analyses.

Domains of Teacher Knowledge

Shulman (1986) argued persuasively for the importance of attending to teachers' knowledge in studies of teaching. He introduced to the educational research community a heuristic model specifying domains that comprise the professional knowledge base of teaching. Four of these knowledge domains are particularly relevant to teachers' instructional practices: subject-matter knowledge, general pedagogical knowledge, pedagogical content knowledge, and knowledge of students. Our work encompasses these domains, although in a different configuration. We consider knowledge of subject matter as a separate domain; however, we integrate the other three domains into a single broader category that we have labeled "mathematics-specific pedagogy." In addition, because a situative perspective on cognition indicates that teachers' professional identities play an important role in their instructional decisions and classroom practice (Greeno, 1998; Greeno & MMAP, 1998) we consider identity as a component of professional expertise.

Thus, within our research project, consideration of prospective teachers' knowledge and beliefs is organized around three domains that are particularly relevant to instructional practices: mathematics subject matter, mathematics-specific pedagogy, and professional identity. Within each domain we explore dimensions of central relevance to teaching in a reform-based context, and we examine teachers' knowledge and beliefs as they are situated—or enacted—in each participant's developing practice. In this article, we focus on one dimension of mathematics subject matter—proof, and two dimensions of mathematics-specific pedagogy—tasks and discourse.

<u>Mathematics Content</u>

Our interest in preservice teachers' beliefs about and knowledge of mathematics stems in part from the research literature that suggests that preservice teachers do not have the mathematics knowledge necessary to enact reform-based images of teaching (Ball, 1990, 1991; Ball & Mosenthal, 1990; Cooney, 1985). We have chosen three mathematics content areas as foci for our project: proof, functions/transformations, and rates. We refer to these areas as "mathematics domain slices." In addition, we look at proof in a broader sense as we explore how students and teachers justify and explain their thinking; in this sense, proof and justification span all three domain slices. These domain slices meet several criteria for important mathematics content to examine in research on reform-based mathematics education (Ball, personal communication, 1997; Wilcox et al., 1990). Proof, functions/transformations, and rates, are (1) taught by beginning teachers, (2) focal to all curricula and students, (3) integral to conceptions of mathematical literacy, (4) important to advanced work in mathematics, (5) prominent in any school of thought about good teaching of mathematics, (6) traditionally difficult for students to learn and for teachers to teach effectively, (7) comprised of skills and concepts and can be located along some continuum involving both, and (8) a target of reform. In this article, we focus primarily on proof and justification.

Proof is central to the discipline of mathematics, and proving is central to the practice of the discipline. Many mathematicians and mathematics educators view proof from a similar perspective, namely, as a social process engaged in by members of a community of mathematical practice, a process by which "mathematics [grows] through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations" (Lakatos, 1976, p. 5). Proponents of this view have described proof as "a debating forum" (Davis, 1986, p. 352), "a form of discourse" (Wheeler, 1990, p. 3), "a social construct and a product of mathematical discourse" (Richards, 1991, p. 23), "a justification arising from social interactions" (Balacheff, 1991, p. 93), and as "an essentially public activity" (Bell, 1976, p. 24).

Yet, the place of proof in secondary school mathematics has historically been peripheral at best, usually limited to the domain of Euclidean geometry. Current reform efforts, however, call upon mathematics teachers to provide all students with rich opportunities and experiences with proof throughout secondary school mathematics curricula—opportunities and experiences that reflect the nature and role of proof in the discipline of mathematics. For example, in the first draft of *Principles and Standards for School*

Mathematics (NCTM, 1998)—the forthcoming revision of NCTM's influential Standards documents (NCTM, 1989, 1991, 1995)—a separate standard on reasoning and proof recommends that in grades pre-K to 12:

> Mathematics instructional programs should focus on learning to reason and construct proofs as part of understanding mathematics so that all students: recognize reasoning and proof as essential and powerful parts of mathematics; make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs; [and] select and use various types of reasoning and methods of proof as appropriate (p. 80).

Are secondary school mathematics teachers prepared to enact these recommendations in their instructional practices? To date, little research has focused on practicing teachers' conceptions of proof. Researchers have examined students' conceptions of proof (e.g., Balacheff, 1991; Bell, 1976; Maher & Martino, 1996), prospective elementary teachers' conceptions of proof (e.g., Martin & Harel, 1989; Simon & Blume, 1996), and undergraduate mathematics majors' conceptions of proof (e.g., Harel & Sowder, 1998). Further, much of the previous research has focused primarily on individuals' understandings of proof and methods of proof. The conceptual framework developed for this study is broader, adressing both the *nature* of proof, and the *role* of proof in the discipline and the classroom. The *nature* of proof refers to the explanations, justifications, and elaborations that serve to make a conjecture more convincing and accurate in the face of refutations (i.e., counterexamples) posed by potential doubters (Lakatos, 1976). The *role* of proof refers to a proof's potential to establish both *that* some assertion is true and *why* it is true (Hanna, 1990, 1991; Hersch, 1993). These two areas capture aspects of teachers' conceptions of proof that are essential for understanding their implementation (or lack thereof) of reform recommendations concerning proof in school mathematics.

Mathematics-Specific Pedagogy

Our decision to integrate general pedagogical knowledge, pedagogical content knowledge, and knowledge of students into the single construct called "mathematics-specific pedagogy" has its roots, in part, in the situative perspective that knowledge is inseparable from the physical and social contexts in which it develops and is used. Subject matter is arguably the most salient feature of the contexts in which secondary teachers work. Their thinking and learning about teaching is situated within the discipline of mathematics; their pedagogical knowledge is embedded in the context of the mathematics classroom (McLaughlin & Talbert, 1993; Wineburg & Grossman, 1998). Therefore, general pedagogical knowledge and knowledge about students, while theoretically distinct from pedagogical content knowledge, are inseparable in practice. In this paper, we focus on two aspects of mathematics-specific pedagogy that have been highlighted by the mathematics educational community as central to reform-based teaching—selection and use of mathematical tasks and classroom mathematical discourse (Clarke, 1994, NCTM, 1991).

Mathematical tasks. The questions, problems, exercises, constructions, applications, projects and investigations in which students engage constitute the "intellectual contexts for [their] mathematical development" (NCTM, 1991, p. 20). Tasks provide the stimulus for students' work in classrooms, and they "convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). As a result, they are an important influence on the mathematics learning that occurs in school.

One of the teacher's primary responsibilities is to select and develop worthwhile tasks—tasks that are "generative, rich with mathematical possibility and opportunity," and contain "hooks that connect the child's world with particular mathematical ideas and ways of thinking" (Ball, 1993). Worthwhile tasks contain important mathematical ideas, represent concepts and procedures, foster skill development as well as problem solving and reasoning, and help students to make connections among mathematical ideas and with real-world applications (NCTM, 1991). Further, they "lend themselves to multiple solution methods, frequently involve multiple representations, and usually require students to justify, conjecture, and interpret" (Silver & Smith, 1996, p. 24). It is through engaging in such tasks that students gain access to the phenomena of mathematics (Nespor, 1994) and come to understand and appreciate what doing mathematics entails. The selection of tasks is complex—teachers must take into account specific students' knowledge and interests, what is known about the ways in which students learn particular mathematical ideas, and common student confusions and misconceptions about those ideas.

Mathematical discourse. Although discourse has always played a central role in teaching and learning, it is currently receiving increased attention in the mathematics education community. Scholars are arguing for—and reform initiatives are underscoring—the importance of teachers and students engaging in oral and written discourse that fosters students' understanding of mathematics (Ball, 1991; Hiebert, 1992; NCTM 1991, 1998; Peressini & Knuth, 1998). These calls for more meaningful discourse in mathematics classrooms are grounded in research demonstrating the social nature of learning mathematics (Cobb, Boufi, McClain, & Whitenack, 1997) and a vision of school mathematical practice that reflects the essence of mathematical practice within the discipline (Lampert 1990). It is through participation in classroom discourse that students become initiated into the community of mathematical practice (Lo, Wheatley, & Smith, 1994). Mathematical discourse in the classroom provides an arena in which the students learn how to represent mathematics through thinking, talking, agreeing, and disagreeing about mathematics, rather than learning from the talk (Lave & Wenger, 1991).

Cobb and his colleagues describe two distinct classroom traditions—the school mathematics tradition and the inquiry mathematics tradition—in which the norms and expectations negotiated by students and teachers are characterized, in part, by distinct differences in patterns of discourse (Cobb, Wood & Yackel, 1993). Discourse within the school mathematics tradition often follows an elicitation pattern in which the solution is the driving force (Voigt, 1995). Typical interactions can be characterized by a three-part process comprised of initiation, reply, and evaluation (IRE, [Mehan, 1979]). The process begins with the teacher posing a known-information question. A student responds, attempting to provide the expected answer, and the teacher follows by evaluating the response. If the student response is incorrect, the teacher continues to call on other students until the desired response is given. This type of interaction promotes "dialogues that typically degenerate into social guessing games when teachers attempt to steer or funnel students to a procedure or answer they have in mind all along" (Cobb et al., 1993, p. 93).

In contrast, student explanations are the driving force for discourse in the inquiry mathematics tradition (Voigt, 1995). Interaction typically begins through information-seeking questions that require students to explain how they interpreted and solved tasks, and that expect students' original contributions (Cobb et al., 1993). Students' responses are evaluated in terms of established sociomathematical norms, such as what counts as an acceptable mathematical explanation and justification (Yackel & Cobb, 1996). To facilitate this type of discourse, a teacher must be skillful at posing questions that challenge student thinking, listening carefully to students' ideas, rephrasing students' explanations in terms that are mathematically more sophisticated, deciding when to provide information, and orchestrating class discussions to ensure participation by all students (Cobb et al., 1991).

The Interdependence of Task, Discourse, and Proof

In sum, we envision the act of teaching mathematics to consist, in part, of: (1) selecting and developing worthwhile tasks which have the potential to immerse students in significant mathematics content, and (2) orchestrating classroom discourse focused on mathematical thinking, reasoning, and communication. These two aspects of teachers' work are clearly interdependent—it is around worthwhile tasks that discourse in the inquiry mathematics tradition will be centered. As Driver, Asoko, Leach, Mortimer, and Scott (1994) noted, "a social perspective on learning in classrooms recognizes that an important way in which novices are introduced to a community of knowledge is through discourse in the context of relevant tasks" (p. 9). Further, classroom discourse that focuses on what counts as an acceptable mathematical explanation and justification engages students in the crucial mathematical notion of proof.

Situating the Incident in Practice: Ms. Savant's Mini-Unit on Circles

Ms. Savant student taught at "Cumulus High School." The school opened 3 years prior to the study in a predominantly white, rapidly-growing upper middle class suburb. Enrollment was 1600 students in grades 9 through 12. In the classroom featured in the opening vignette, there were 11 boys and 13 girls (18 Caucasion, 2 African American, 2 Asian, and 2 exchange students from Mexico who did not speak English). About one-third of the students were repeating Algebra I; two-thirds were taking the class for the first time.

Ms. Savant was responsible for developing the 4-day mini-unit on circles during which the Pizza Slices activity took place. We observed the first two days of this unit. As Ms. Savant explained during our interview at the beginning of this data collection cycle, one central goal for the mini-unit was "to get the kids to recognize that π isn't just a number, that it's a relationship . . . between circumference and diameter." She planned to focus on π and circumference on the first day, area of a circle on the second day, and arc lengths and area of circle sectors on subsequent days.

Ms. Savant began the first day of the unit with a whole-class review of definitions of key terms associated with circles (e.g., circumference, radius, diameter, arc), eliciting responses in what might be characterized as a typical IRE pattern (Mehan, 1979). Following this review, students worked in groups to measure the diameters (d) and circumferences (C) of circular objects in the classroom, and to compute the ratio C/d for each object. They then computed the average ratio for each group and for the whole class, and noted that the class ratio, 3.163, was very close to π. Ms. Savant commented to the class, "You know what that means? That means you guys were incredibly precise in the way you measured. That's very cool."

Two subsequent lesson segments focused on the meaning of π. In the first, for example, Ms. Savant projected a computer image of a circle she had created using *Geometer's Sketchpad*[2], on the whiteboard at the front of the room. The class watched as one student "dragged" one edge of the circle, changing the displayed values of its diameter and circumference. The displayed ratio of these two measurements, however, remained constant. It was this relationship ($C=2\pi r$) that she asked students to recall during the Pizza Slices activity, which began the next day's class.

Ms. Savant's Practice: The View from our Conceptual Framework

In this section we discuss Ms. Savant's beliefs, knowledge, and practice with respect to three components of our conceptual framework: proof, task, and discourse. We consider how a situative perspective on cognition helps us to understand the relationship between Ms. Savant's knowledge and beliefs in these areas and the practices we observed in the mini-unit on circles.

Proof

Ms. Savant referred to the Pizza Slices illustration of the equation for area of a circle as a proof, both in describing her plans for the activity ("My plan is to...see if they can prove it using those wedges.") and at the end of the activity ("You guys have just proved that the area of a circle is πr^2.") Her use of this language led us to wonder about her understanding of the nature of proof and its role in school mathematics. In an interview later that day, the researcher asked Ms. Savant if she thought the Pizza Slices illustration was a proof. She responded, "Yes, I do.... It's not a formal proof in that we sit down and we write it all out. But certainly it's a way to prove to yourself, 'Hey, look, this actually really does work." She commented further, "My idea of a formal proof is when you actually sit down, and you use symbolic language with English language, and you give either a paragraph or a step-by-step proof that's very logical." She contrasted this

2 *Geometer's Sketchpad*® (Key Curriculum Press) is a dynamic geometry construction program.

notion of a formal proof with that of an informal proof, or convincing argument, which is acceptable and appropriate in a high school mathematics classroom.

Ms. Savant also made this distinction between formal and informal proofs throughout a series of interviews focused on her knowledge of mathematics content and mathematics-specific pedagogy (See Appendix A). She described formal proof as the structured symbolic presentations acceptable to mathematicians, whereas informal proofs are ways to explain why something is true. “I think convincing [informal proof] is different from formal proof. But, in the vernacular English it would prove to the kids that it works.” The validity of convincing arguments depends “both on the person who’s writing the proof and the person who’s reading the proof [who] have to agree on what the generalized terms mean.”

Later in the interview conducted on the day of the Pizza Slices activity, the researcher asked Ms. Savant to rate this proof on a 1-4 scale, with 4 being most valid (as she had done with several other proofs in the mathematics content interviews). She responded:

> Boy, it’s really hard to say. I guess I’m still stuck in the mathematician mode, where a formal proof is a formal proof,...and that’s a 4. I haven’t convinced my brain yet that proofs without words are OK. So, I’d probably give it somewhere between a 2 and a 3. But, if kids came up with it on their own, I’d give them a 3 1/2 on it. ... I wouldn’t give a 4 to kids who knew how to do formal proofs. Does that make sense? If they didn’t know how to do formal proofs, and we had talked about ways to prove things, I’d probably say that given what they know, if they could reconstruct this and explain it well, I’d probably give it a 4.

This response confirms Ms. Savant’s situated conception of proof: that the audience is as important as the speaker in determining whether an argument is convincing, and that whether or not an argument is convincing determines its usefulness as proof in a secondary mathematics classroom. This conception, in turn, helps explain Ms. Savant’s use of the Pizza Slices activity.

<u>Instructional Tasks</u>

Ms. Savant incorporated a number of activities, using a variety of instructional formats, into her mini-unit on circles. She explained, “A lot of the activities I’m using today and tomorrow are actual activities from the math methods class.” For these activities, including Measuring Circular Objects and Pizza

Slices, "I wanted to try and see how they work in a high school classroom as opposed to a math methods classroom." Her idea for the computer demonstration came from work she did with *Geometer's Sketchpad* in both her non-Euclidean geometry and mathematics methods classes. This software was important in helping Ms. Savant learn to enjoy geometry. "Using *Sketchpad* is a really neat experience for me. It helps me see how things work together, and it helps me feel comfortable with geometry. I think it's neat to be able to see what happens without having to laboriously plot points and measure things." She hoped that it might have a similar impact on her students.

Ms. Savant believes in the importance of actively engaging students in learning experiences. In an interview prior to student teaching, she attributed her commitment to hands-on activities, in part, to the field experience associated with her mathematics methods course. There, she saw for herself "how excited kids get when they're actually hands on and trying to figure things out," and she realized that "the students gained far more understanding and joy from the exercise by working it out themselves than by having me give them the correct answer."

Thus, although the Pizza Slices activity was presented as a teacher demonstration in her mathematics methods class, Ms. Savant decided to have the students do the activity themselves. A situative perspective helps us to understand this decision. As Ms. Savant explained "These kids are much more engaged when they have something that they're doing.... If they're manipulating things and then I start talking about it on the board, they've actually done the manipulations so they have something concrete that they can relate the abstract to. I think that's really important."

Classroom Discourse

Whereas Ms. Savant appeared confident in her mathematics knowledge and her selection of instructional activities, she was less confident in her ability to engage the class in mathematical discourse. Ms. Savant expressed concerns about this aspect of her teaching prior to student teaching. For example, she commented during an interview that one of her earlier field experience mentors "probably is a lot like I am and was raised a lot like I was. I think he talks too much to the kids and doesn't allow them enough open-ended questions." She noticed that the students seemed to tune out when this teacher talked, and she worried that she might have a similar impact on her students. Our observations during the mini-unit confirmed that Ms. Savant remained at the center of most whole-class discussions.

Ms. Savant wanted to improve this aspect of her teaching. When Kate Rockford (her cooperating teacher) asked her to identify areas of her teaching in which she would like to show improvement, Ms. Savant replied “I would like to have more mathematical discussion. I think I’m still giving them more than they need. I’m not giving as much as I used to, and I’m asking them to talk more, but I’d like to see the kids talking more.” Focusing on a similar issue, she commented to the researcher, “When I see [Ms. Rockford] work with the kids, she has such a natural style that the kids start to feel really comfortable with each other. And they have this conversation back and forth which is really neat to have. It’s happened once for me in a really amazing way in one of the classes, when the kids started talking to each other instead of looking at me and answering questions. I haven’t found that comfort zone yet for myself.... What is too little control? What’s too much control? ... So that’s what I’m looking for, to try and get the kids to start on their own in stating questions about math.” A situative perspective sheds light on the challenges Ms. Savant faced in trying to take on a new role in mathematical discussions–as a leader in her own classroom, rather than a participant in the mathematics methods course. We explore this notion further in our conclusions.

Situating Ms. Savant’s Learning in Teacher Education Experiences

In this section we explore the impact of two key teacher education experiences—the university mathematics methods course and public school student teaching—on Ms. Savant’s learning to teach.

Ms. Savant’s Mathematics Methods Class

Ms. Savant’s mathematics methods course, taken the semester prior to student teaching, was one of two primary sites for reform of teacher preparation for secondary mathematics teachers at Mountain State College (MSC)[3]. It is a 3-credit Mathematics Department course with a 60-hour field placement shared with a general methods course offered by the Teacher Education Department. The instructor articulated the course goals this way:

> One of the goals is to connect students' mathematical knowledge that they've been accruing as math majors...with the mathematics that they'll be teaching as secondary math teachers.

> So the class includes lots of opportunities to do 6-12 level problems and think about the mathematics behind the problems.... The second goal is to get them to think about student learning of mathematical ideas as a complex process of constructing understanding....The third goal is to think about the act of teaching as all the things that go into getting students to interact with mathematical ideas and to do mathematics in legitimate ways...to get them to think more about discourse and conversations as being authentic mathematical activities in addition to formal presentations of mathematical ideas.

In an interview at the end of the course, Ms. Savant described what she thought were the goals of the course, “I don't think that his objectives were necessarily to give us particulars so much as to make us consider broad goals in teaching. In particular, how are we going to ask questions of our students and how are we going to get our students to not be afraid to participate in mathematics.”

Our analysis of the videotape record of this course identified two layers of influences on Ms. Savant’s teaching. The first layer is characteristic of most methods classes, and it was noted by Ms. Savant herself: the course provided students with a large collection of tasks. For example, activities in which students compared the circumferences of circles to their diameters, and watched the instructor model how to find the area of a circle by cutting it into wedges and rearranging the wedges to form a parallelogram, played out over a sequence of three consecutive classes sessions. Ms. Savant developed her Measuring Circular Objects and Pizza Slices activities from these experiences.

The second, deeper layer of influence was due to the content and sequence of tasks, and the ways in which students engaged in these tasks. We encountered a consistent theme throughout the course: tasks were chosen and sequenced, and classroom discourse was structured, to portray proof broadly as explanation and justification and to encourage students to reflect on how these specific features of mathematics and mathematics-specific pedagogy might influence their own teaching. We highlight one set of activities that illustrates this theme.

In a sequence of homework assignments, in-class activities, and computer-lab explorations that spanned several weeks, students constructed geometric objects like the bisector of an angle, the

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[3] MSC (a pseudonym) is a four year urban commuter college with a large teacher preparation program. The 17,000

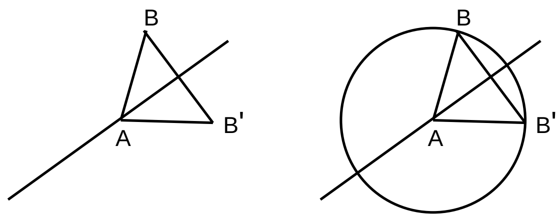
perpendicular bisector of a line segment, and a circle inscribed in a triangle—and explained why these constructions work. They compared the properties, strengths, weaknesses, and learning opportunities provided by construction tools like the classic compass-and-straightedge, the *Geometer's Sketchpad* software, and the MIRA, a translucent plastic device that allows its user to exploit reflective symmetry to perform constructions. They also read and discussed a research study (Schoenfeld, 1988) which concluded that high school geometry students' knowledge of constructions and proofs is procedural and unconnected. The instructor posed the following task at the beginning of the final class in this sequence:

> Place a point A on a sheet of paper. Draw a line through A, and draw a point B about an inch from the line. Use the line you drew as a line of reflection, and use the MIRA to find the image of the point B. Use at least five other lines of reflection through A to find image points for B. If you were to find the set of images of B across all possible lines of reflection through A, what figure would this set of points form? Why?

After about 8 minutes of work in small groups, the students agreed that the figure is a circle centered at A with radius AB. Ms. Savant volunteered to explain why. She stepped to the board and said:

> Well, we used side-angle-side. We said, "Let's assume this is our line of symmetry here [drawing point A and a line through A]." Then we took a perpendicular from here [point B] straight down through here [the line of symmetry] to here, and we called this B'. Now assume, even though my drawing is really bad, that these [the angles formed by the line of symmetry and the line from B to B'] are 90 degree angles. And we know that this [the segment from A to the intersection with the perpendicular] is always going to be the same distance. We know that since these two points are reflected about this line, then by the definition of reflection this has to be equal to this [marking the two segments that connect B through the line of reflection to B']. Since these are 90 degree angles we have [pointing in turn] side, angle, side. Then, corresponding parts of corresponding figures are going to be the same, so these two [pointing to AB and AB'] have to be the same. And you can do this all the way around the circle.

undergraduates have a mean age of 27 years.



The students agreed that Ms. Savant's proof that point B and every image of B will be the same distance from A explained why this figure had to be a circle.

In the end of course interview, Ms. Savant described what she would say to someone who was about to take the course, "I'd tell them to expect a lot of proof work and to expect to have some pretty serious discussions about what and how you should be teaching mathematics. In our groups we talked a lot about how things worked and how we might use those things as teachers. And even when we were discussing how we attempted to solve problems, we were in essence talking about how students might attempt to solve problems." The course was not what she had expected. She had heard that other methods courses were "very procedural oriented as opposed to content oriented." This course, in contrast, was about "learning how to encourage kids and get them to start making intellectual forays into math as opposed to just, passively, memorizing the book and regurgitating it on the test; teaching kids how to talk about the math that they're doing in their head or how to write about the math that they're doing in their head."

At the conclusion of this interview, Ms. Savant reflected on the course, her field experiences thus far, and what she thought she still had to learn.

> What we've been taught to do is still an abstraction for me in the regular everyday classroom because I didn't see it in practice. … I learned a lot from my teachers. I learned some practical things that will be helpful. I learned some class management things that will be helpful. I learned some things that I don't want to do, and that's always helpful. But I didn't learn how to put into practice the reform mathematics style that we're being asked to teach.

Ms. Savant's comments, combined with our analysis of the mathematics methods class sessions, suggest that the course not only provided the students with a variety of resources to assist them in their teaching, but also encouraged them to think in new ways about mathematics learning and teaching, and specifically about the concepts of proof, task and discourse. Ms. Savant took these new resources and ideas to her student teaching experience, hoping to learn how to put them into practice in a high school mathematics classroom.

Ms. Savant's Student teaching Placement

Kate Rockford, Ms. Savant cooperating teacher at Cumulus High School is a National Board for Professional Teaching Standards certified secondary mathematics teacher and recognized instructional leader throughout the district. This placement was arranged by Ms. Savant's mathematics methods instructor. When asked about her role in Ms. Savant's student teaching, Ms. Rockford said, "My role isn't to make her be like me." She elaborated:

> My role isn't necessarily to tell her the right answer or how she should be. I really don't think that I have the ideal answer in mind. I think that that's going to come through with her own perspective...when she reflects, so my goal is to be a cheerleader and to be a resource.… I said to her, 'My job is to give you a room with students in it and get out of your way and give you positive feedback.' …We all need to have confidence. I can already tell that she's going to be her own worst critic; she doesn't need me to point out anything. I'm just going to be…sharing with her some things I've tried and letting her try her own. She has plenty of confidence to take risks on her own.… I want to try to help her think of things from a new vantage point.

Ms. Rockford and Ms. Savant met daily to plan and reflect on Ms. Savant's classes. They spoke frequently about Ms. Savant's desire to encourage students to participate more in class discussions. Ms. Rockford saw improvement in this aspect of Ms. Savant's teaching and was confident that she would continue to improve. "One thing about students talking is that it takes a real paradigm change for most math teachers to really value that. I remember when [Ms. Savant] first came to me and she said, 'I'm a really good math teacher one-on-one with kids, but I don't know how I'll be 30-to-1. I've been taught traditionally that the teacher talks, and I need you to teach me how to get kids to talk.' So I think that's something that she feels she's a beginner in." Ms. Rockford emphasized, "I think that she's way ahead of a lot of the math teachers in this building.... So, you've got to put it in perspective." And, "You get better at [fostering student talk] when you trust that math learning will happen without the teacher being involved. I really think that...more and more, as she watches me, she's going to begin to try some techniques.... And I think that's something that I can help her grow with, throughout the next couple of months."

Thus, Ms. Savant had ample opportunities to try activities and instructional approaches she thought would be beneficial for her students. Ms. Rockford was there to provide coaching, feedback and support. Ms. Rockford's "vantage point" embraced the vision of a mathematics classroom that engaged students in conceptually meaningful mathematical activity. These were just the kinds of opportunities Ms. Savant had hoped to have in her student teaching experience, in order to learn how to put reform-based mathematical ideas into practice.

Discussion

What can we learn from Ms. Savant's experiences about the process of learning to teach and about the role of teacher education in that process? How does a situative perspective on cognition help us to understand the nature of, and influences on, teacher learning? We address these questions below.

Teacher education did make a difference in Ms. Savant's development as a teacher. Her mathematics methods course provided a large collection of *tasks* compatible with a reform vision of teaching—tasks that encourage multiple representations; lend themselves to multiple solution strategies; and actively involve students in making conjectures, providing justifications and explanations, and drawing connections. During student teaching, Ms. Savant began to incorporate these tasks into her own teaching repertoire, adapting them to her particular students and curriculum. The mathematics methods course also engaged pre-service teachers in *discourse* characterized by discussions in which mathematical explanations and justifications played a central role. Ms. Savant found it difficult to engage students in such inquiry-oriented discourse in her own classroom. She had developed a commitment to this type of discourse, however, and her ability to engage students in mathematical discussions improved during the student teaching experience. Finally, *proof*—in both the formal and informal senses—played a central role in the mathematics methods course. Although Ms. Savant was, at times, "still stuck in the mathematician mode, where a formal proof is a formal proof," her conception of informal proof focused more on the explanatory power of the argument. The mathematics methods instructor wanted his students to understand the mathematical reasons underlying formulas, procedures and rules—a goal that Ms. Savant carried with her into her own classroom instruction.

Why is it that the set of learning opportunities provided Ms. Savant by this particular teacher education program made a difference? A situative perspective addresses this question by focusing attention on the relationship between learning and the settings in which it occurs. By considering what ideas and practices Ms. Savant brought from one setting to another, a situative perspective directed our attention to issues of compatibility of goals and visions across the various settings of teacher education.

The ultimate goals for novice teachers—that they assume the professional responsibilities of a teacher and teach competently—are undoubtedly shared by members of the multiple communities involved in teacher education. It may well be the case, however, that specific conceptions of what it means to be a comptent teacher are different, and at times even incompatible, across these communities. Further, members of these diverse communities may hold different ideas about how a person learns to teach and the kinds of experiences that best facilitate such learning (Grossman et al., 1999). To the extent that conceptions of professional competence or the practices that guide teachers' development toward these conceptions are incompatible across communities, these communities are likely to compete rather than support each other in their impact on novice teachers' learning. In their analysis of the breakdown of the apprenticeship model in reform-based teacher education, Sykes and Bird (1992) suggested that such competition, with its limiting effect on teacher learning, is particularly likely when one (or more) of the teacher education communities is promoting reform-based educational ideas and practices:

> Finally, the situated cognition perspective draws on the image of apprenticeship in a guild or a professional community as a powerful form of learning. But this image requires a stable, satisfactory practice that the novice can join. If the aim of teacher education is a reformed practice that is not readily available, and if there is no reinforcing culture to support such practice, then the basic imagery of apprenticeship seems to break down. Teachers' knowledge is situated, but this truism creates a puzzle for reform. Through what activities and situations do teachers learn new practices that may not be routinely reinforced in the work setting? (p. 501)

In Ms. Savant's teacher education program, a great deal of effort went into ensuring that two key settings—the mathematics methods course and the student teaching placements—were compatible along several crucial dimensions. The mathematics methods instructor, who was responsible for arranging student teaching placements, consciously sought highly qualified teachers who shared the program's vision of mathematics education reform, and whose classrooms fit reform-based images. Further, he sought teachers who, like Ms. Rockford, believed that it is important to allow student teachers to try out their own ideas, rather than pushing them to "be like me." Ms. Savant noted at the end of the mathematics methods course, "I learned a lot.... But I didn't learn how to put into practice the reform mathematics style that we're being asked to teach." The student teaching placements were selected to provide a stable, supportive setting in which student teachers could attempt to put their reform-based ideas into practice.

In contrast, Ms. Savant heard and invoked different ideas about the nature and role of proof as she participated in different teacher education settings. When she talked about doing proofs for mathematicians, she stressed that proofs should be formal arguments that use structured, symbolic language. But when she talked about the role of proof in her mathematics methods course and student teaching, she discussed proof as an informal sense-making process that shows why something is true. Our data indicate that while both of these conceptions of proof developed as she progressed through her teacher education program, only one had a substantial influence on Ms. Savant's growth as a classroom teacher. When her teacher preparation experiences focused on the role of proof in the explanation and justification of mathematical ideas—as in her mathematics methods course—these experiences influenced her subsequent practice in important ways. However, when her experiences portrayed proof more formally—as in most of her mathematics content courses—these experiences seemed to contribute little to her practice. This pattern, viewed through a situative perspective, may help explain why researchers find little relationship between increased content preparation and improved teaching competence.

A situative perspective also helps us to understand why some aspects of reform-based pedagogy are more easily learned than others. Here, the explanation centers on the question of how transportable ideas and practices learned as a student in the university setting are to the novice teacher's role as a teacher in the K-12 field setting. Ms. Savant discovered that she could take specific tasks introduced in her mathematics methods course and successfully adapt them to her own classroom and students. These tasks were easily transportable across situations, despite the fact that she was a student in one situation and a teacher in the other. Such was not the case with discourse patterns. Ms. Savant attempted to incorporate inquiry-based discourse modeled by her mathematics methods instructor into her own teaching, and she solicited Ms. Rockford's assistance in this effort. She did show some improvement, according to Ms. Rockford, but she still considered herself a novice in this area. When we observed her student teaching, her struggle with the goal of fostering discourse was apparent. Despite compatible goals and visions, her mathematics methods and student teaching situations differed in one crucial way: Ms. Savant was responsible for initiating and establishing expectations for inquiry-based discourse in her own classes, as opposed to simply participating in such discourse as a student. Her teacher education program provided Ms. Savant with little knowledge of how to manage this responsibility; thus, she found it much more difficult to transport discourse patterns, than instructional tasks, into her teaching practice. When we shared this manuscript with Ms. Savant, she offered another insight about the idea of transportability:

> . . . mathematics can be a solitary occupation in which you are the "leader" of your own work, engaging in discousre with yourself about how to solve a problem. Thus the change to a leader with respect to content knowledge, in the classroom, is not so much a change but an expansion of the role. However, the change to a leader of interactive discourse between people is a true change from "leader" of internal discourse with yourself.

The learning-to-teach literature led us to consider the possibility that personal factors such as limitations in Ms. Savant's professional knowledge and skills caused the difficulty she experienced transporting discourse patterns from the university setting into her teaching practice. We explored two factors that typically present problems for novice teachers: subject matter knowledge and classroom management knowledge and skills. A number of studies suggest that teachers with greater subject matter knowledge tend to emphasize the conceptual, problem-solving, and inquiry aspects of their subjects whereas less knowledgeable teachers tend to emphasize facts, rules, and procedures (Borko & Putnam, 1996). Focusing specifically on classroom discourse, Carlsen (1987, 1991) compared secondary science student teachers' teaching of topics for which they had high and low subject matter knowledge. When teaching topics for which they had high subject matter knowledge, they asked fewer questions but a greater proportion of higher order questions, and they tended to talk less of the time and for shorter periods of time. Students talked more, asked more questions, volunteered to speak more, and spoke in longer discourse sequences. Another especially salient task for the novice teacher is acquiring the knowledge and skills for managing a classroom. Indeed, some researchers have argued that novice teachers must become competent in the skills of classroom management before they can successfully turn their attention to other aspects of their teaching (Berliner, 1989; Kagan, 1992). Although suggested by the literature, limitations in subject matter knowledge and classroom management knowledge and skills do not appear to be reasonable explanations for Ms. Savant's difficulties engaging the class in mathematical discourse. Our series of mathematics content interviews indicated that her knowledge was strong in all three mathematics domain slices that are the focus of our project: proof, functions/transformations, and rates. And, unlike many student teachers, Ms. Savant did not mention classroom management as a concern in either her conferences with Ms. Rockford or her interviews about student teaching. Nor did our classroom observations reveal management problems.

Our research suggests that experiences in both university courses and K-12 field placements are crucial for preparing teachers to teach according to the mathematics education community's vision of reformed practice. The lens offered by a situative perspective on cognition leads us to conclude that compatibility of these settings on several key dimensions is essential for the settings to reinforce each other's messages, and thus work in conjunction, rather than in opposition, to prepare reform-minded teachers. When the situations of teacher education share conceptions of teacher learning and a vision of reformed practice, teacher education does make a difference in preparing reform-oriented educators to join the profession.

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Appendix A

LTSM Data Collected from and about "Audrey"

Fall 1997/Spring 1998

- Intellectual Profile interview
- Mathematics Content I interview
- Foundations of Geometry course:
  - videotape and field notes of all classes
  - end-of-course interview
  - interview with Instructor
- Pedagogy I interview
- Identity I interview, metaphor task
- Identity II interview, metaphor update

Fall 1998

- Mathematics Methods course:
  - videotape and field notes of all classes;
  - end-of course interview;
  - pre- and post-interviews with Instructor
- Mathematics methods course field experience:
  - videotape and field notes of teaching on one occasion;
  - end-of-experience interview;
  - interview with cooperating teacher;
- Identity III interview

Spring 1999

- Student Teaching: three cycles of data collection
  - Videotape and field notes of teaching
  - Field notes and audiotape of meetings with cooperating teacher, college supervisor
  - Daily interviews with student-teacher
  - Interviews with cooperating teacher, college supervisor
- Audiotape and field notes of student teaching seminars
- End-of-Student teaching/Seminar interview
- End-of-Teacher-Education-program interview

Summer 1999

- Mathematics Content II interview
- Pedagogy II interview
- Identity IV interview