

**Instructions:** Work the following problems; give your reasoning and show your supporting calculations. Do not give decimal approximations unless the nature of a problem requires them. Your paper is due at 2:50 pm.

1. Evaluate the following limits. Use the Limit Laws. You need not mention the Limit Laws explicitly, but you must show the calculations they lead you to.

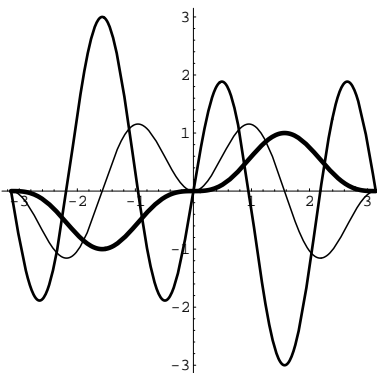
(a) 
$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 4x + 3}$$

(b) 
$$\lim_{x \rightarrow \infty} \frac{(5 - x)(10 + 8x)}{(3 - 3x)(3 + 10x)}$$

2. A rock is thrown off of a 100-meter cliff with an upward velocity of 50 m/s. As a result, its height (above the ground at the base of the cliff) after  $t$  seconds is given by the formula  $h(t) = 100 + 50t - 5t^2$ .

- (a) What is its height after 7 seconds?  
 (b) What is its average velocity over the first seven seconds?  
 (c) What is its velocity after 7 seconds?  
 (d) What is its velocity when it hits the ground at the base of the cliff?

3. Here is a graph showing three functions—a skinny one, a middle-weight one, and a fat one—on the same pair of axes:



One of the curves is  $f$ , one is  $f'$ , and one is  $f''$ . Explain which is which and how you know.

4. Find  $f'(x)$  if

(a)  $f(x) = 3x^2 - 4x + 5$

(b)  $f(x) = \frac{x^2 - 2\sqrt{x}}{x^{1/3}}$

5. Find  $f'(x)$  if

(a)  $f(x) = (3x^2 - x + 1)^2(5x + 4)^{12}$

(b)  $f(x) = \frac{a}{x^{10}} + \sin^3 bx$ , where  $a$  and  $b$  are fixed but unspecified constants.

6. A function  $f$  is given by

$$f(x) = \begin{cases} 2cx + 2 & ; \quad x \leq 3 \\ 3 - cx & ; \quad 3 < x. \end{cases}$$

For what values of the constant  $c$  is  $f$  continuous on  $(-\infty, \infty)$ ? Be sure to give your reasoning.

7. (a) Use the definition of the derivative to find  $f'(x)$  if  $f(x) = \sqrt{2x}$ .

(b) Use the derivative you calculated in part (a) of this problem to write equations for the lines tangent to the curve  $y = \sqrt{2x}$  at  $x = 1$ , at  $x = 2$ , and at  $x = 8$ .

8. Let  $F$  be the function given by

$$F(x) = f(x)g(x),$$

where  $f$  and  $g$  are functions for which  $f'(x)$  and  $g'(x)$  are both defined for all real values of  $x$ .

(a) What is  $F'(x)$ ?

(b) Derive the formula you have given in part 8a.

Complete solutions to the exam problems will be available from the course web-site later this evening.