MTH 1410, Calculus I—Exam III **April 22, 2009**

Instructions: Work the following problems *on your own paper*; give your reasoning and show your supporting calculations. Do not give decimal approximations unless a problem requires you to do so. Your exam is due at 2:50 pm.

1. Find the limits:

   (a) $\lim_{x \to -1} \dfrac{x + x^2}{\ln(2 + x)}$

   (b) $\lim_{x \to 0} \dfrac{xe^{-2x}}{\pi e^{2x} - \pi}$

2. Find the absolute maximum and the absolute minimum for the function

$$f(x) = 2x^3 - 3x^2 - 12x + 20$$

   on the interval $[-3, 3]$.

3. Let F be the function given by

$$F(x) = (x - 1)^2(x + 1)^3.$$

   Then, in fully factored form,

$$F'(x) = (x - 1)(x + 1)^2(5x - 1)$$

   and, also in fully factored form,

$$F''(x) = 20(x + 1)\left[x - \frac{1}{5}\left(1 - \sqrt{6}\right)\right]\left[x - \frac{1}{5}\left(1 + \sqrt{6}\right)\right].$$

   Use this information to determine the intervals where F is increasing, the intervals where F is decreasing, the intervals where F is concave upward, and the intervals where F is concave downward. What are the critical numbers of F? What is the nature of each of the critical points (local maximum, local minimum, or neither)? *Give your reasoning.*

4. Let f be the function given by

$$f(x) = \begin{cases} x^2 + 2x, & x \le 2 \\ ax^2 + b, & x > 2. \end{cases}$$

   (a) What condition must the constants a and b satisfy if f is to be a continuous function?

   (b) Find all pairs of values for a and b which make the function f a differentiable function.

5. Find the points on the ellipse $x^2 + 4y^2 = 4$ whose distance from the point $(1, 0)$ is minimal.

6. Murgatroyd was driving his car toward an intersection at 60 miles per hour. A police cruiser was approaching the same intersection but on the cross-street (which is at right angles to the road that Muratroyd is on), at 50 miles per hour. When both cars were a quarter of a mile from the intersection, a police officer in the cruiser pointed a radar gun at Murgatroyd and measured the speed at which the two cars were approaching each other. What did she get?