MTH 1410, Calculus I—Final Exam **May 11, 2009**

Instructions: Work the following problems *on your own paper*; give your reasoning and show your supporting calculations. Do not give decimal approximations unless a problem requires you to do so. Your exam is due at 2:50 pm.

1. (a) *Use the definition of the derivative* to find $f'(x)$ if $f(x) = 1/\sqrt{x}$.

   (b) Use the derivative you calculated in part (a) of this problem to write equations for the lines tangent to the curve $y = 1/\sqrt{x}$ at $x = 1$, at $x = 4$, and at $x = 9$.

2. Evaluate the following definite integrals. Give all of your reasoning.

   (a) $\displaystyle\int_3^5 (3x^2 - 24x + 54)\,dx$

   (b) $\displaystyle\int_0^3 3t\sqrt{9 - t^2}\,dt$

3. Let F be the function given by

$$F(x) = (x-2)^2(x+3)^3.$$

   Then, in fully factored form,

$$F'(x) = 5x(x-2)(x+3)^2$$

   and, also in fully factored form,

$$F''(x) = 20(x+3)\left[x - \sqrt{\frac{3}{2}}\right]\left[x + \sqrt{\frac{3}{2}}\right].$$

   Use this information to determine the intervals where F is increasing, the intervals where F is decreasing, the intervals where F is concave upward, and the intervals where F is concave downward. What are the critical numbers of F? What is the nature of each of the critical points (local maximum, local minimum, or neither)? *Give your reasoning.*

4. Suppose that $f(2) = 2$, $f(4) = 4$, $f'(2) = 4$, $f'(4) = -2$, $g(2) = 4$, $g(4) = 2$, $g'(2) = -6$, and $g'(4) = -8$.

   (a) Find $F(4)$ and $F'(4)$, where $F(x) = \dfrac{f(x)}{g(x)}$.

   (b) Find $G(2)$ and $G'(2)$, where $G(x) = g[2f(x)]$.

   (c) Find $H(2)$ and $H'(2)$, where $H(x) = g[f(x^2)]$.

5. (a) Show that the point $(3, 2)$ lies on the curve given by the equation

$$x^3 - 5x^2y^3 + 8y^4 + 205 = 0.$$

   (b) If x and y are related by the equation, $x^3 - 5x^2y^3 + 8y^4 + 205 = 0$, find the value of y' at $(3, 2)$.

   (c) Show how to use the results of parts (a) and (b) of this problem to find an approximate value for y near 2 when $x = 74/25$.

6. Find the points on the hyperbola $4y^2 - x^2 = 1$ whose distance from the point $(5, 0)$ is minimal.