MTH 3440, Partial Differential Equations—Exam I **March 4, 2009**

Instructions: Work the following problems; give your reasoning and show your supporting calculations. Your paper is due at 12:50 pm.

1. Classify each of the following partial differential equations as elliptic, parabolic, or hyperbolic. *Give reasons for your choices.*

   (a) $u_{xy} = 0.$

   (b) $u_{xx} + u_{xy} + u_{yy} = 2x.$

   (c) $u_{xx} - u_{xy} + u_{yy} = 2u.$

   (d) $u_{xx} - 2u_{xy} + u_{yy} = u_y.$

   (e) $u_{xx} - u_{yy} - u_y = 0.$

2. Which of the following are solutions of the partial differential equation $u_t = \alpha^2 u_{xx}$? (Note: No boundary conditions; no initial condition.) *Give your reasoning.*

   (a) $u(x,t) = e^{-\lambda^2\alpha^2 t}(\cos\lambda x - \sin\lambda x)$, where λ is an arbitrary constant.

   (b) $u(x,t) = 3x - 2.$

   (c) $u(x,t) = 2e^{\lambda^2\alpha^2 t}\tan 2\lambda x$, where λ is an arbitrary constant.

   (d) $u(x,t) = 4\alpha^2 e^{4\alpha^2 t}\cosh 2x.$

3. Suppose that $u(x,y)$ and $v(x,y)$ are indefinitely differentiable functions that meet the conditions

$$u_x(x,y) = v_y(x,y)$$
$$u_y(x,y) = -v_x(x,y)$$

   everywhere in the plane. Show that $u(x,y)$ and $v(x,y)$ must both be solutions of the partial differential equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

   throughout the plane.

4. Find the finite Fourier cosine transform on $[0,1]$ of the function f given by $f(x) = x - x^2$.

5. Find the positive eigenvalues and the corresponding eigenfunctions of the Sturm-Liouville problem

$$\begin{aligned} X''(x) + \lambda X(x) &= 0; \\ X(0) &= 0; \\ X'(1) &= 0. \end{aligned}$$

6. Show how to transform the IBVP

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, && 0 < x < 1,\ 0 < t < \infty, \\ u(0,t) &= \cos\pi t, && 0 < t < \infty, \\ u(1,t) &= \sin\pi t, && 0 < t < \infty, \\ u(x,0) &= x, && 0 < x < 1, \end{aligned}$$

   into an IBVP with homogeneous boundary conditions. (*Do not* attempt to solve the transformed problem.)