MTH 3240, Partial Differential Equations—Problem Set 2 **Due Feb. 25, 2009**

Instructions: Work the following problems and submit your solutions at the beginning of class on Monday, Feb. 25, 2009. You may use any resources you like, but the solutions you submit must be solutions that you, yourself, have written from the understanding that you have gained from the resources you use. If I have doubts about your understanding of a solution you have presented, I may ask you to come to my office and elaborate it orally.

1. Let f be a function which is defined and indefinitely differentiable on the interval $[0, \infty)$. The *Laplace transform*, $\mathcal{L}[f]$, of f is the function given by

$$\mathcal{L}[f](s) = \int_0^\infty f(t) e^{-st}\, dt, \tag{1}$$

provided that the improper integral converges. Show how to derive the formulae:

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0), \tag{2}$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0), \text{ and} \tag{3}$$

$$\mathcal{L}[f^{(3)}](s) = s^3\mathcal{L}[f](s) - s^2 f(0) - sf'(0) - f''(0). \tag{4}$$

You may assume that all of the Laplace transforms you encounter converge and that

$$\lim_{t\to\infty} e^{-st} f^{(k)}(t) = 0,$$

for all $k \geq 0$, at least when $s > 0$.

2. Use the method of Lesson 6, Farlow, to show how to transform the IBVP

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1,\ 0 < t < \infty, \tag{5}$$

$$u(0,t) = \cos \pi t, \qquad 0 < t < \infty, \tag{6}$$

$$u(1,t) = \sin \pi t, \qquad 0 < t < \infty, \tag{7}$$

$$u(x,0) = x, \qquad 0 < x < 1, \tag{8}$$

into an IBVP with homogeneous boundary conditions.

3. Find the solution for the IBVP of Problem 2, above.